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Investigation of the natural convection boundary condition in microfabricated structures

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Abstract

Heat loss through surrounding air has an important thermal effect on microfabricated structures. This effect is generally modeled as a natural convection boundary condition. However, the correct procedure for the determination of the convective coefficient (h) at microscales continues to be debated. In this paper, a microheater is fabricated on a suspended thin film membrane. The natural convection on the microheater is investigated using 3-omega measurements and complex analytical modeling. It is found that the value of h that fits experimental data should have an apparently larger value than that at larger scales; however, it is also shown that the increased h is actually contributed by heat conduction instead of heat convection. A method of determining the correct h that can be used for microfabricated structures is proposed by using the heat conduction shape factor.

Keywords: Microscale natural convection; Convective coefficient; 3-omega measurement; Micro-Electrical–Mechanical-System (MEMS) device; Micro fabricated structure; Microscale heat transfer

1. Introduction

Natural convection is a frequently encountered boundary condition in the thermal design, testing and modeling of Micro-Electrical–Mechanical-System (MEMS) devices. The small device scales and the increased demands on accurate thermal control require the convective coefficient (h) to be precisely determined. This requirement is particularly important for those MEMS devices directly exposed to air and with apparent temperature difference from the ambient.

Although natural convection at larger length scales is a mature discipline, the effect of scaling below 1 mm on the natural convective coefficient (h) is still a topic of debate. The complexity lies in the interplay of the very thin boundary layers and the relative change of importance of driving forces at the microscale. Peirs et al. [1] have proposed a scaling law for natural convective coefficient and suggested $h \sim 100\ \mathrm{W/m^2\,K}$ for air when the scale is less than 100 μm, which is 5–10 times larger than that at macroscales. This enhanced heat transfer coefficient is explained in terms of better convection heat transfer due to compressed boundary layers at very small scales. On the other hand, Guo et al. [2] argued that natural convection should be less significant at the microscale because buoyancy, the driving force for natural convection becomes very small. Experiments have shown that the best value of h that fits the measured results for microfabricated heated surfaces is generally larger than macroscale h, such as $35\ \mathrm{W/m^2\,K}$ [3,4]. However, the mechanism behind this elevated h is not well understood.

In this paper, a microheater is fabricated on a silicon nitride membrane and the natural convection on the microheater is precisely characterized using the 3-omega method (see [5,6] for reviews of the 3-omega method), with the heater oriented at different angles to the gravitational field. The measured results are compared with comprehensive modeling. Natural convection on the microheater is found to be very feeble, even though the heater is in a large space; however, h can still be useful as a

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Nomenclature

a	half width of the suspended membrane = 450 μm	
b	half width of the aluminum heater line = 5 μm	
f	frequency of the driven current	Hz
h	convective coefficient	$W/m^2\,K$
h_r	radiative coefficient	$W/m^2\,K$
i	imaginary unit	
k	thermal conductivity	$W/m\,K$
l	the length of the microheater device = 1800 μm	
n	summation index $1, 2, \ldots$	
P	applied electrical power	W
R	electrical resistance	Ω
R_0	the electrical resistance of the microheater	Ω
S	shape factor	m
T	temperature	K
t	thickness of the suspended membrane = 2.1 μm	
V	voltage along the microheater	V
$V_{3\omega}$	voltage component at 3ω	V
x, y	Cartesian coordinates	μm
Z	thermal impedance	K/W
Greek symbols		
α	thermal diffusivity	m^2/s
$\Delta T_{2\omega}$	the temperature oscillation on the microheater	K
η	eigenvalue in y-coordinate	m^{-1}
ε	emissivity	
θ	relative temperature oscillation	K
Λ	mean free path	m
ω	angular frequency of the driven current	rad/s
σ	Stefan–Boltzmann constant $=5.67\times10^{-8}\ W/m^2\,K^4$	
ξ	eigenvalue in x-coordinate	m^{-1}
Subscripts		
a	air	
m	membrane	

parameter for thermal MEMS design if it is related to the effective heat conduction to the surrounding medium. Based on these findings, a method of estimating the effective h for MEMS design is proposed by using the concept of shape factors of heat conduction.

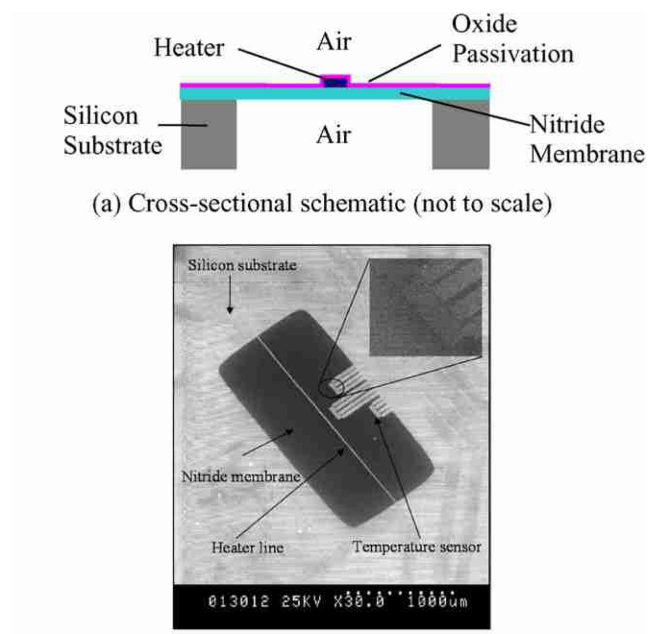
2. Experimental measurements

Microscale natural convention to air is measured using a microfabricated heater structure, which consists of an aluminum heater line (1800 μm long and 10 μm wide) on a silicon nitride membrane (1800 μm long, 900 μm wide, 1.7 μm thick, passivated with 0.4 μm silicon oxide), as shown in Fig. 1. Heat generated from the heater can either be conducted through the membrane to the silicon substrate, or lost through the surrounding air if the structure is exposed to atmosphere. Thermal isolation of the membrane provides good sensitivity to any heat loss from the microheater.

The device is fabricated at the Stanford nanofabrication facility (SNF). The 1.7 μm thick layer of silicon nitride is first deposited on a single-crystal double-side polished ⟨100⟩ silicon wafer in a low-pressure chemical vapor deposition (LPCVD) furnace at 850 °C. After sputtering 0.2 μm of aluminum and photolithography, aluminum is dry-etched to define a heater line bisecting the membrane and then passivated from the environment by depositing a 0.4 μm thick film of silicon dioxide in an LPCVD furnace at 400 °C. Pad etch (a mixture of acetic acid and ammonium fluoride) is used to gain electrical access to metal contact pads connecting to the heater. Silicon nitride deposited on the backside of the wafers is stripped. The membrane is finally formed by photolithography, front-to-backside alignment and Deep Reactive Ion Etching (DRIE) of the silicon substrate from the backside, during which the silicon nitride serves as an etch stop.

The fabricated microheater structure is calibrated in a cryogenic chamber. The cryostat has an internal 25 W resistive heater and a platinum thermometer, both connected to a temperature controller. The temperature of the chamber can be controlled with accuracy $\leqslant\pm0.023$ K and long-term stability $\leqslant\pm0.01$ K. The pressure during calibration is less than 4 Pa (30 mtorr). The calibrated temperature dependence of the heater resistance is given in Fig. 2, which is a strait line with a slope $\mathrm{d}R/\mathrm{d}T = 0.11996\ \Omega/K$ and a linearity of 0.999992.

The measurements are performed in the same chamber using the 3-omega method, either in vacuum ($\leqslant$4 Pa) or open to atmosphere. Driven by the sine out of a lock-in amplifier at an



(a) Cross-sectional schematic (not to scale)

(b) Top-view scanning electronic microscope (SEM) image

Fig. 1. Microfabricated heater structure for the measurements of natural convection at microscales.

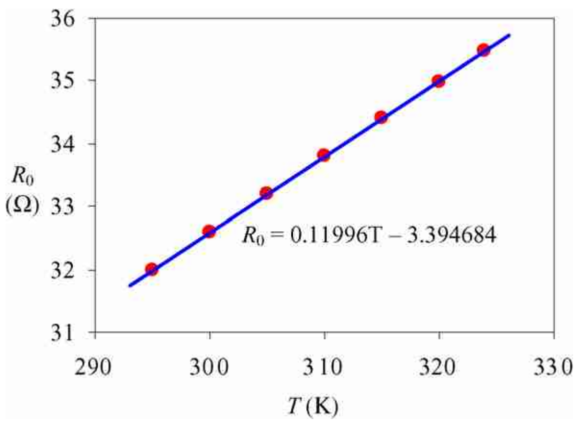


Fig. 2. Calibrated microheater resistance versus temperature.

angular frequency of ω, the microheater generates Joule heat and produces a temperature oscillation $\Delta T_{2\omega}$ at frequency 2ω. It is noted that the electrical resistance of the microheater is proportional to its temperature, and thus the electrical resistance is also modulated at 2ω. With the applied current at ω, the voltage drop along the microheater thus contains a modulated component at 3ω. This 3ω voltage component, $V_{3\omega}$, measured using the lock-in amplifier, is related to the temperature oscillation $\Delta T_{2\omega}$ as [5]:

$$\Delta T_{2\omega} = 2\frac{\mathrm{d}T}{\mathrm{d}R}\frac{R_0}{V}V_{3\omega} \tag{1}$$

where V is the applied voltage and R_0 is the electrical resistance along the microheater.

3. Analytical modeling

In order to determine the exact temperature solution for this problem, the thermal impedance of the membrane is first derived. Since the membrane is very thin (1.7 μm silicon nitride and 0.4 μm silicon oxide) and the microheater is patterned on it symmetrically, the heat conduction in the membrane can be simplified into a 2-dimensional problem, as depicted in Fig. 3. If we consider the heat conduction within the membrane alone (i.e. in vacuum), the steady-state periodic temperature oscillation θ (a complex variable depicting both the amplitude of temperature oscillation relative to the ambient T_0 and the phase

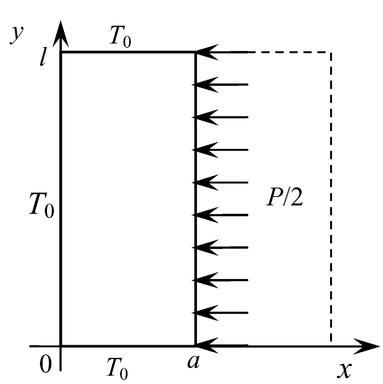


Fig. 3. Thermal analysis of the microfabricated membrane.

angle relative to the driving current) can be described using the frequency-domain heat conduction equation

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = \frac{\mathrm{i}\omega}{\alpha_\mathrm{m}}\theta \tag{2}$$

with boundary conditions

$$\theta(x,0) = \theta(x,l) = \theta(0,y) = 0 \tag{3}$$

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=a} = \frac{P}{2k_\mathrm{m}tl} \tag{4}$$

where i is the imaginary unit, α_m, k_m, a, t, l are the thermal diffusivity, thermal conductivity, half width, thickness, and length of the membrane, respectively. P is the applied power and ω is the angular frequency of the heat flux. This PDE system can be solved using the method of separation of variables. The solution is

$$\theta = \frac{P}{\pi k_\mathrm{m}tl}\sum_{n=1}^{\infty}\frac{1+(-1)^{n-1}}{n\xi_n\cosh(\xi_n a)}\sinh(\xi_n x)\sin(\eta_n y) \tag{5}$$

with eigenvalues

$$\xi_n = \sqrt{\eta_n^2 + \frac{\mathrm{i}\omega}{\alpha_\mathrm{m}}} \tag{6}$$

$$\eta_n = n\pi/l \tag{7}$$

The temperature oscillation measured on the microheater is an averaged value along the y-direction at $x = a$, i.e.

$$\Delta T_{2\omega} = \frac{1}{l}\int_0^l \theta(a,y) = \frac{2P}{\pi k_\mathrm{m}tl^2}\sum_{n=1}^{\infty}\frac{1+(-1)^{n-1}}{n\xi_n\eta_n}\tanh(\xi_n a) \tag{8}$$

Thus, the thermal impedance of half of the membrane can be given by

$$Z_\mathrm{m}(\omega) = (P/2)/\Delta T_{2\omega} = \left[\frac{1}{\pi k_\mathrm{m}tl^2}\sum_{n=1}^{\infty}\frac{1+(-1)^{n-1}}{n\xi_n\eta_n}\tanh(\xi_n a)\right]^{-1} \tag{9}$$

The heat loss through the surrounding air can be due to either convection or conduction. If we can introduce an effective convective coefficient h averaged over the entire membrane, the convective thermal impedance can be given by

$$Z_\mathrm{a} = 1/(2hal) \tag{10}$$

If we model the heat loss through the surrounding air as pure heat conduction, after using the analytical solution for 3ω measurements of a microheater on a semi-infinite substrate [5], the thermal impedance can be given by

$$Z_\mathrm{a} = \left\{\frac{1}{\pi k_\mathrm{a}l}\left[\frac{1}{2}\ln\left(\frac{\alpha_\mathrm{a}}{\omega b^2}\right) + \ln 2 - 0.5772 - \frac{\mathrm{i}\pi}{4}\right]\right\}^{-1} \tag{11}$$

where α_a, k_a are the thermal diffusivity and thermal conductivity of the surrounding air, respectively, and b is the half width of the microheater.

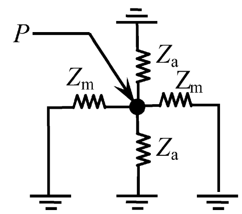


Fig. 4. Equivalent thermal circuit of the microfabricated structure exposed to air.

The thermal impedances of the membrane and the surrounding air are in parallel, as depict in Fig. 4. Thus, the averaged temperature oscillation along the microheater, when the micro structure is exposed to air, can be calculated by

$$\Delta T_{2\omega} = 2P\left(Z_{\mathrm{m}}^{-1} + Z_{\mathrm{a}}^{-1}\right) \tag{12}$$

4. Results and discussion

The measured temperature oscillation in the microheater in vacuum is shown in Fig. 5 as a function of frequency. Dots are experimental data, and curves are calculated from Eq. (8). Power input for all measurements is maintained at $P = 4.28 \pm 0.02$ mW. Since heat conduction through the suspended membrane is the dominant heat loss mode (radiation is small due to small temperature differences around room temperature), the amplitude of the temperature oscillations at a given power input is only a function of the thermal properties of the suspended membrane. From top to bottom, the curves represent membrane thermal conductivities ranging from 0.5 W/m K, to 0.94 W/m K, 1.5 W/m K and 2.0 W/m K. The value that fits the experimental data best is 0.94 W/m K, which is in good agreement with the thermal conductivity value obtained from steady-state measurements using the integrated temperature sensors on the membrane [4].

After the structure is exposed to atmosphere, the heat loss through the surrounding air reduces the amplitude of the temperature oscillations, as shown in Fig. 5. These reduced temperature oscillations can be modeled using Eq. (12) based on the fact that the heat loss through the surrounding air is in parallel to heat conduction through the suspended membrane. Heat loss through the surrounding air is first modeled using natural convection, as expected from heat transfer at large scales. To do so, Eq. (10) is adapted to model the thermal impedance of air. The value of h that best fits the experimental data for the entire area of membrane is 30.4 W/m^2 K. The fitted temperature oscillations are given by the solid blue curve in Fig. 6.[2] If we calculate h using the area of the microheater only, the value of h could be more than 2700 W/m^2 K. These h values are substantially bigger than 5–10 W/m^2 K that are generally encountered for natural convection in air at large scales. These data seem to indicate that natural convection is enhanced at small scales.

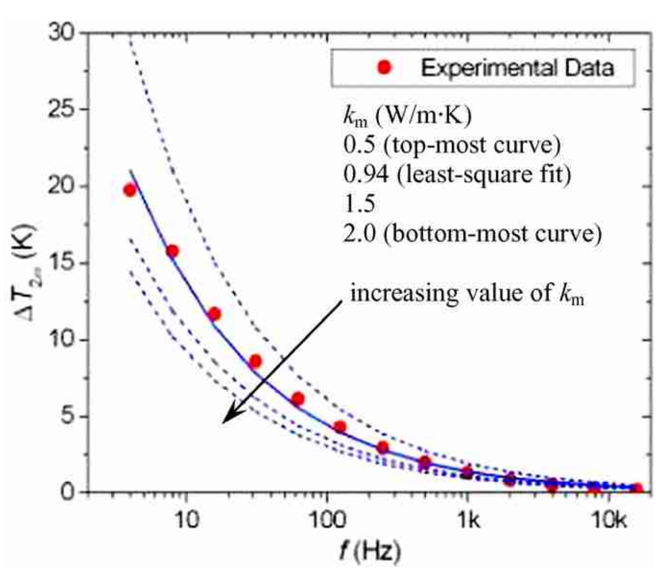


Fig. 5. Temperature oscillations of the microheater in vacuum driven by a sine current at the frequency of f.

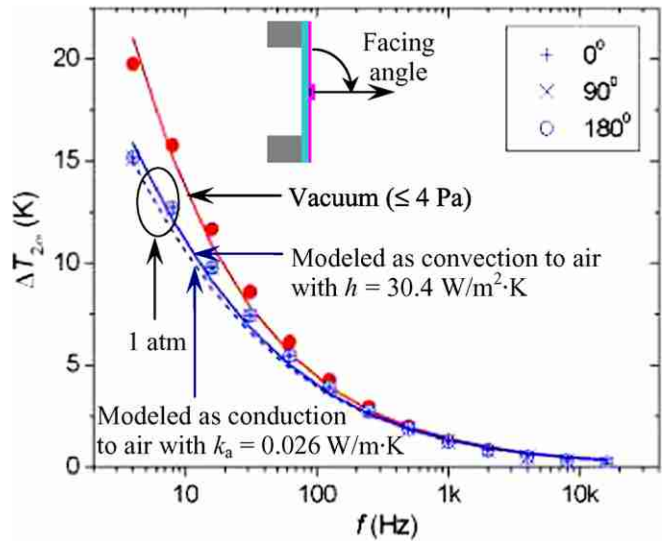


Fig. 6. Temperature oscillations of the microheater in atmosphere with variable orientations, and compared with the data obtained in vacuum.

However, it also should be noted that in Fig. 6, the measured temperature oscillations are almost independent of the heater orientation to the gravitational field. This cannot be explained if the natural convection is the dominant heat transfer mode to air. Another possible heat transfer mode is heat conduction. Assuming pure heat conduction without any air flow, the thermal impedance of the air can be modeled using Eq. (11). This model yields the dashed blue curve in Fig. 6 with $k_{\mathrm{a}} = 0.026$ W/m K. The good agreement between the model and the experimental data suggests that the heat loss to air from the microheater is actually dominated by heat conduction and that natural convection is negligible, although the heater is facing a large space. Therefore, even though h can still be phenomenally used for microstructures, the value of h can be estimated just based on the analysis of heat conduction to surrounding air (or fluid), i.e.

$$h = k_a S/A \tag{13}$$

[2] For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

where k_a is the thermal conductivity of surrounding air, A is the "convective" heat transfer area and S is the heat conduction shape factor [7]. For this particular problem, $S = 1/(k_a Z_a)$ with Z_a determined by Eq. (11). It is no surprise that h is larger than that for macro scales. This is because heat conduction is relatively more significant for small devices, owing to the reduced thermal capacitance and the increased surface to volume ratio. The air thermal conductivity k_a may decrease due to the Knudsen effect when the device characteristic length (L) close to the air molecule mean free path (Λ), which is about tens of nanometers at room temperature and one atmosphere. In this case, k_a should be modified by a factor of $1/(1 + Kn)$, where $Kn = \Lambda/L$ is Knudsen number. Besides, at high temperature, radiation correction on Eq. (13) should be added. The correction factor is $h_r = 4\varepsilon\sigma T^3$, where ε is the emissivity of the microdevice and $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2\,K^4}$ is the Stefan–Boltzmann constant.

5. Conclusion

Heat loss from a micro fabricated heater to its surrounding air is experimentally measured using the 3ω method and analytically modeled based on two heat transfer modes: convection and conduction, respectively. Unlike at large scales, it is found that for a microscale heater, the natural convection from the heater to surrounding air is negligible and the heat loss is dominated by heat conduction. However, natural convection boundary condition can still be phenomenally used with an increased h. Since heat loss due to conduction is relatively more significant for microstructures, a practical method to estimate h for MEMS device design is to use the heat conduction shape factor as proposed by Eq. (13).

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