Measurement of ballistic phonon conduction near hotspots in silicon

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The Fourier law for lattice heat conduction fails when the source of heat is small compared to the phonon mean free path. We provide experimental evidence for this effect using heating and electrical-resistance thermometry along a doped region in a suspended silicon membrane. The data are consistent with a closed-form two-fluid phonon conduction model, which accounts for the severe departure from equilibrium at the hotspot. The temperature rise exceeds predictions based on the Fourier law by 60% when the phonon mean free path is a factor of 30 larger than the resistor thickness. This work is improving the constitutive modeling of heat flow in deep-submicron transistors. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1371536]

Transistor scaling has reduced the channel length below the phonon mean free path in silicon, which is approximately 300 nm at 300 K.[1,2] The hotspot within a transistor, where electron energy transfer to the lattice is most intense, can be 10 nm thick.[2] The small size of the phonon source compared to the phonon mean free path may yield dramatically larger transistor temperatures than those predicted using the Fourier law of heat diffusion.[2,3] This results from the ballistic, or nonlocal, nature of phonon conduction and can be predicted using the Boltzmann transport equation. Ballistic phonon transport in silicon films was investigated through the large measured reductions in the lateral thermal conductivity compared to the bulk value.[1,4] However, the effect of ballistic transport near a small heat source, which has greater implications for transistors, has not been experimentally investigated.

We report measurements of ballistic phonon conduction near a doped resistor thermometer in silicon, shown in Fig. 1, which approximates the smaller phonon source in a transistor. To induce ballistic transport, we perform measurements at 100–200 K where the phonon mean free path ranges between 2 and 10 μm. Joule heating in the doped resistor induces a temperature rise in the membrane structure. The thermal conductivity and mean free path in the silicon membrane are determined from the temperature rises of the parallel aluminum bridges A, B, and C, which serve as electrical resistance thermometers. Ballistic phonon transport is observed through the departure of the doped resistor temperature from predictions based on the Fourier law. The membrane geometry increases the thermal resistance of the experimental region and reduces the experimental uncertainty.

The resistor is fabricated using boron implantation at 40 keV with a dose of 5×10^{15} cm^{-2} through a patterned oxide mask in a silicon-on-insulator (SOI) film. Our simulations show that the effect of implant damage on thermal conductivity of the resistor is negligible for these conditions. The current along the resistor is confined to the *p*-type region by reverse biasing the junction formed with the *n*-type substrate. The patterned aluminum bridges are far from the doped resistor compared to the phonon mean free path, such that their temperatures yield the lateral thermal conductivity of the silicon membrane independent of nonlocal transport near the doped resistor. The membrane is formed by plasma etching the underlying silicon. The structure is attached within a ceramic die package, electrically bonded, and mounted in a cryostat.

The experimental procedure begins with calibration of the electrical resistances of the aluminum bridges and doped resistor between 100 and 300 K. The calibration uses four point probe measurements at very low electrical currents, such that the temperature rise due to Joule heating is negligible. After calibration, the cryostat is fixed at a targeted base temperature and the current along the doped resistor is increased from the 0.1 mA calibration value to a value near 1.2 mA. The current generates approximately 7.5 mW/m, referenced per unit length normal to Fig. 1, heating the entire membrane and raising the doped resistor temperature by approximately 3 K. The current along the doped resistor, and therefore the peak temperature rise, is limited by the breakdown voltage of the junction formed with the *n*-type substrate. The steady-state resistance changes of the doped resistor and the three aluminum bridges are recorded, and the

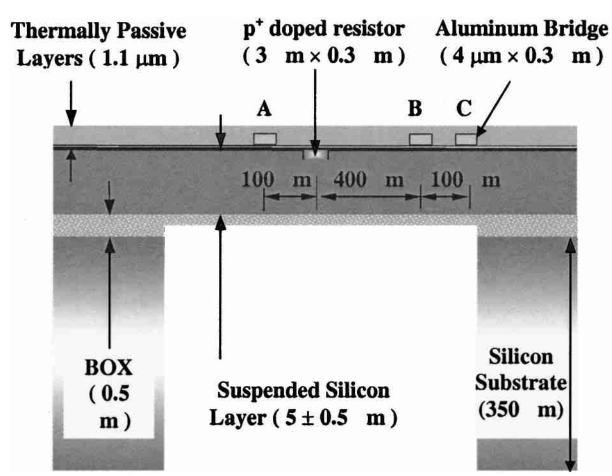

FIG. 1. Cross section of the experimental structure, which is micromachined from a SOI wafer. The doped resistor in the silicon membrane is used for heating and thermometry.

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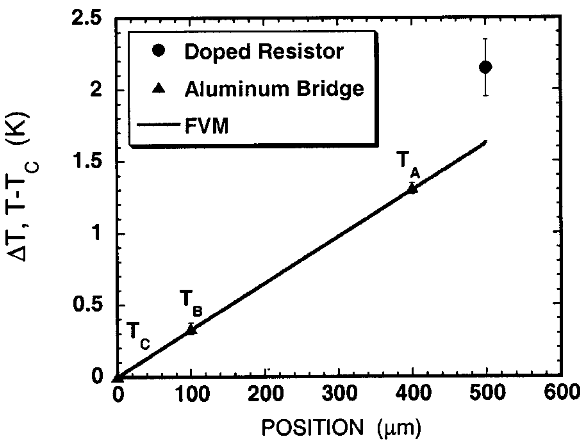

FIG. 2. Data and predictions for the temperature rise along the membrane. The temperature rises are referenced with respect to the temperature measured by the aluminum resistor *C*. Because of symmetry, the aluminum bridges provide three temperatures along what can be viewed as a single heat flow path for determining the membrane thermal conductivity.

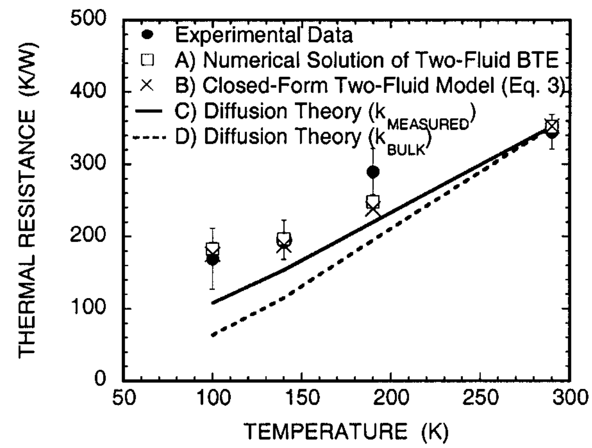

FIG. 3. Data and predictions for the membrane thermal resistance as a function of temperature. Prediction *A* is a numerical solution to the phonon Boltzmann transport equation with a simplified two-fluid model. *B* is a closed-form correction to diffusion theory based on the two-fluid phonon model. *C* and *D*, are based on diffusion theory, using the measured membrane thermal conductivity and the bulk thermal conductivity, respectively.

temperature rises of the four thermometers are calculated from the calibration results.

Figure 2 shows predictions and data for the temperature distribution along the membrane. The data from the three aluminum thermometers lie almost perfectly on a line, as expected from the heat diffusion equation. The membrane thermal conductivity is calculated using

$$k=\frac{q'}{d}\left(\frac{dT}{dx}\right)^{-1}, \tag{1}$$

where q' is the heat flux per unit length into Fig. 1, d is the membrane thickness, and dT/dx is the slope determined from the aluminum bridge temperatures. The thermal conductivities observed here are smaller than those of bulk samples owing to phonon-boundary scattering and are consistent with previous measurements and predictions for SOI wafers.[4,5] Near-ballistic phonon conduction along a membrane is relatively well understood[6,7] compared to the ballistic phonon transport near the doped resistor, which approximates a hotspot and is the focus of the current study.

Figure 2 shows that the temperature rise at the doped resistor exceeds the value predicted using the heat diffusion equation and the Fourier law. This can be qualitatively shown by comparing the doped resistor temperature with a linear extrapolation of the aluminum temperatures. The peak temperature from the linear extrapolation agrees within 0.5% with a finite volume method numerical solution to the two-dimensional heat conduction equation.

The temperature rise of the doped resistor in Fig. 2 exceeds the predictions based on the Fourier law at a base temperature of 190 K by a margin exceeding the experimental uncertainty. If this effect results from ballistic phonon transport, we expect the temperature margin to become larger for longer mean free paths. The mean free path Λ is calculated from the membrane thermal conductivity using a relationship based on the Boltzmann equation, which has been confirmed by experimental data for silicon.[4,5] Since the base temperature controls the phonon mean free path in the present experiment, it is useful to compare theoretical predictions of thermal resistance with data over a broad range of temperatures.

Figure 3 provides this comparison by plotting the thermal resistance of the membrane at different base temperatures. The thermal resistance is the ratio of the temperature rise between the doped resistor and the membrane edge to the applied power. Two of the prediction curves are based on the Fourier law, applied using Eq. (1). The lowest resistance is predicted using the bulk conductivity of silicon[8] since the mean free path and thermal conductivity are largest. In the membrane, the thermal conductivity is increasingly reduced as the temperature is lowered by phonon scattering on the membrane boundaries.[4] The resistance is better predicted using the thermal conductivity measured in the membrane, as described earlier in this letter. The difference between the data and the diffusion theory prediction using the measured membrane thermal conductivity results from ballistic phonon transport near the doped resistor. The relative increase in thermal resistance varies between 56% at 100 K, where the mean free path is approximately 10 μm, to less than 1% at 290 K, where the phonon mean free path is approximately 300 nm.

The phonon mean free path is required for the two additional predictions in Fig. 3, which consider the ballistic nature of phonon transport. To properly account for the non-equilibrium phonon transport near the heat source, the phonon dispersion relationship must be considered. The phonons emitted by Joule heating are not necessarily of the same energy, polarization, or mode as the phonons dominating thermal conduction. For electrons of sufficient energy, the energy transfer from electrons to the lattice can be dominated by scattering on optical phonons,[9,10] which have negligible group velocity and contribute little to thermal conduction. A two-fluid model[1,11] accounts for this situation by separating phonons into propagating and reservoir groups, with temperatures T_P and T_R. The average lattice temperature T_L, is weighted by the fractional contributions of the two groups to the heat capacity. As the size of a heat source is reduced, the propagating phonons have less opportunity for scattering with the warmer reservoir phonons as they traverse the heated region. This results in a thermal resistance between the lattice and the propagating phonons at the hotspot, Eq. (2),[12] which must be added to the Fourier law resistance

$$R_{\mathrm{th}} \equiv \frac{(T_L - T_P)_{\max}}{q'} = \frac{\Lambda^2}{3 k_{\mathrm{bulk}} A_{\mathrm{eff}}} \tag{2}$$

where A_{eff} is an effective heat source area, calculated by dividing the maximum volumetric heat generation rate into q'. The heat generation distribution for this calculation is obtained by solving the Poisson and charge continuity equations with a doping profile observed experimentally using secondary ion mass spectroscopy. The expression on the right is derived from the energy equations for the reservoir and propagating groups, which relate their temperatures to the heat generation rate and the phonon mean free path. Equation (2) shows that this resistance grows as the square of the ratio of mean free path to the root of the heat source area. Figure 3 shows that the simple model in Eq. (2) agrees well with a detailed solution to the Boltzmann transport equation applied to the two-fluid phonon system. Both predictions are consistent with the data at 100 and 140 K, where the departure from diffusion theory is substantial. The data at 190 K exceed the predictions substantially, and represent a large departure from any existing theory for reasons that remain unclear.

In this experiment, the nonequilibrium effect increased the total thermal resistance of the membrane by 56% at 100 K. The largest possible impact of the ballistic effect can be estimated using Eq. (2) and a spherical heat source in an infinite medium. In this case the ratio of the thermal resistance using Eq. (2) R_{th} to the thermal resistance using the Fourier law $R_{\mathrm{diffusion}}$ is[12]

$$R_{\mathrm{th}}/R_{\mathrm{diffusion}} = \left(\frac{\Lambda^2}{4\pi r^3 k}\right)\left(\frac{1}{4\pi r k}\right)^{-1} = \frac{\Lambda^2}{r^2}, \tag{3}$$

where r is the effective radius. In a transistor with hotspot dimensions near 30 nm,[2] temperature with phonon mean free path near 300 nm, the spreading resistance is underestimated by two orders of magnitude. This phenomenon may make the temperature rise within the transistor quite important.

This work provides data for the extra thermal resistance near a hotspot resulting from ballistic phonon transport. The data are consistent with the predictions of a two-fluid phonon conduction model except at 190 K, where the data indicate a more dramatic departure from conventional theory. The two-fluid model proposed here vastly simplifies the phonon dispersion relationships by combining phonons into only two separate groups. This work is facilitating the modeling of heat transport in semiconductor devices.

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[1] Y. S. Ju and K. E. Goodson, Appl. Phys. Lett. **74**, 3005 (1999).
[2] P. G. Sverdrup, Y. S. Ju, and K. E. Goodson, ASME J. Heat Transfer **123**, 130 (2001).
[3] G. Chen, 1996, ASME J. Heat Transfer **118**, 539 (1996).
[4] M. Asheghi, Y. K. Leung, S. S. Wong, and K. E. Goodson, Appl. Phys. Lett. **71**, 1798 (1997).
[5] M. Asheghi, M. N. Touzelbaev, Y. K. Leung, S. S. Wong, and K. E. Goodson, ASME J. Heat Transfer **120**, 30 (1998).
[6] E. T. Swartz and R. O. Pohl, Rev. Mod. Phys. **61**, 605 (1989).
[7] H. B. G. Casimir, Physica (Amsterdam) **5**, 495 (1938).
[8] Y. S. Touloukian and E. H. Buyco, *Thermophysical Properties of Matter* (Plenum, New York, 1970).
[9] K. Takeda, K. Sakui, A. Taguchi, and M. Sakata, J. Phys. C **16**, 729 (1983).
[10] A. Majumdar, in *Microscale Energy Transport*, edited by C.-L. Tien *et al.* (Taylor and Francis, New York, 1998).
[11] B. Armstrong, Phys. Rev. B **23**, 883 (1981).
[12] P. G. Sverdrup (2001), Ph.D. thesis, Stanford University.